NONLINEAR MODELS OF CLASSICAL CEPHEIDS ENDOWED WITH TANGLED MAGNETIC FIELDS

RICHARD STOTHERS
NASA Institute for Space Studies, Goddard Space Flight Center

ABSTRACT

The effect of tangled magnetic fields has been included in a new study of full-amplitude models of classical Cepheids. As compared with nonmagnetic models, the magnetic models have longer periods, larger amplitudes, and earlier phases of the small secondary bump that appears on the velocity and light curves. The induced changes of period and of bump phase yield better agreement with observations if Cepheids have normal evolutionary masses. But the predicted amplitudes are larger than those observed; moreover, the inferred value of the mean ratio of magnetic pressure to thermodynamic pressure falls significantly below the value needed to explain the period ratios of the double-mode Cepheids.

Subject headings: stars: Cepheids — stars: interiors — stars: magnetic

I. INTRODUCTION

Magnetic fields of tens to hundreds of gauss have been detected in the atmospheres of several classical Cepheids (Weiss, Dorfi, and Tscharnuter 1981; Borra, Fletcher, and Poeckert 1981). These magnetic fields must extend downward into the interior, where they could be relatively strong and are perhaps fairly chaotic in arrangement. Linear pulsational models of classical Cepheids pervaded by small-scale and randomly oriented magnetic fields reveal that the fundamental pulsation period Π_0 increases, while the period ratios Π_1/Π_0 and Π_2/Π_0 decrease, as a result of the magnetic presence in the star (Stothers 1979b). Since standard Cepheid models, without magnetic fields, prove to have values of Π_0 and Π_1/Π_0 that are too small and too large, respectively, to account for the observed fundamental and first-overtone periods (e.g., Cox 1980), strong magnetic fields may be one way of providing the necessary agreement. Another difficulty with the standard models is that they predict too late a phase of the small secondary bump that appears on the velocity and light curves of many observed Cepheids. Simon and Schmidt (1976) have pointed out that the predicted phase of this bump in standard nonlinear models is closely correlated with the linear period ratio Π_2/Π_0. If this correlation holds for magnetic as well as nonmagnetic models, then the presence of strong magnetic fields may possibly lead to an explanation for the anomalous phase of the bump in observed bump Cepheids (Stothers 1979b).

The purpose of the present paper is to derive actual nonlinear models of classical Cepheids pervaded by well-tangled magnetic fields. These magnetic models turn out, however, to be only partially successful in explaining the observed properties of bump Cepheids, as will be shown in § IV.

II. ASSUMPTIONS

The physical and mathematical assumptions that will be made for the magnetic field $\boldsymbol{H}$ are the same as those described in a previous paper (Stothers 1979a). Briefly, the magnetic field is assumed to be distributed in small, randomly oriented flux tubes, whose hydrodynamical effect is computed by evaluating the gradient of a pseudo-isotropic pressure, $\langle H^2 \rangle / 24\pi$, in the equation of motion, where all quantities are assumed to be averaged over a spherical shell. The magnetic field lines are regarded as being thermodynamically locked into the gas during the course of the pulsations, owing to the high electrical conductivity of the ionized matter.

The necessary equations that contain the magnetic field strength explicitly are

$$\frac{d^2 r}{dt^2} = -\frac{GM(r)}{r^2} - 4\pi r^2 \frac{\partial}{\partial M(r)}\left(P + \frac{\langle H^2 \rangle}{24\pi}\right) \quad (1)$$

and

$$\frac{d\langle H^2 \rangle}{dt} = \frac{4}{3}\frac{\langle H^2 \rangle}{\rho}\frac{d\rho}{dt}, \quad (2)$$

where all the notation is conventional (Stothers 1979a). The energy-conservation law for the star as a whole is

$$L = \int \epsilon \, dM(r) - \frac{d}{dt}(K + W + E_{\text{mag}}), \quad (3)$$

with $K = \int (3/2)P \, dV$, $W = -\int GM(r)r^{-1}dM(r)$, and $E_{\text{mag}} = \int \langle H^2 \rangle/8\pi \, dV$. Only a small number of modifications to the computer program described elsewhere for nonmagnetic stars (Christy 1967; Vemury and Stothers 1978) need to be made here. Observe that equation (2) can be directly integrated, thus becoming $\langle H^2 \rangle/24\pi = C\rho^{4/3}$, where C is a time constant for each

mass shell. The run of C through the model is fixed once and for all from the specified radial distribution of $\langle H^2 \rangle$ in the initial equilibrium model. The magnetic field is assumed to permeate the entire pulsating envelope.

For compatibility with our previous linear work, we assign in the equilibrium model a radial distribution of $\langle H^2 \rangle$ that satisfies either $\langle H^2 \rangle /24\pi = $ constant or $v = $ constant, where

$$v = \langle H^2 \rangle /24\pi P . \tag{4}$$

As in our previous work, we employ deep stellar envelopes, no convection, and two sets of opacities: (1) Cox-Stewart opacities, in the analytic form given by Christy (1966); and (2) Carson opacities (for temperatures greater than $\log T = 3.85$), in standard tabular form (similar to the opacities in Carson, Stothers, and Vemury 1981). The (hydrogen, metals) abundance is taken to be $(X, Z) = (0.700, 0.020)$ for the first set of opacities and $(X, Z) = (0.739, 0.021)$ for the second set. The stellar models have been followed in time until they reach limiting amplitude (in the fundamental mode).

Notation adopted in the present paper includes: K.E., peak kinetic energy; Δ, full (not half) amplitude; *Asymmetry*, time spent on the descending branch of the surface velocity curve divided by time spent on the ascending branch; ϕ, phase after minimum radius of the second (but not necessarily the secondary) bump on the surface velocity curve plus unity; and *Bump*, location of the secondary bump on the descending (D) or ascending (A) branch of the surface velocity curve. The "surface" of the dynamical models is assumed to be the fixed mass layer that, in the equilibrium model, lies at optical depth ~ 0.2. Velocity is measured positively outward. Finally, the period ratios Π_2 /Π_0 have been computed by using linear nonadiabatic theory.

III. A MODEL WITH A UNIFORM MEAN MAGNETIC FIELD

The prototype bump Cepheid model studied by Vemury and Stothers (1978) had $M = 7 \, M_\odot$, $\log (L/L_\odot) = 3.7$, $\log T_e = 3.78$, and $\Pi_0 = 8.72$ days, based on the Carson opacities. This model has been recalculated here by introducing a uniform mean magnetic field with $\langle H^2 \rangle^{1/2} = 300$ gauss and then by following the hydromagnetic oscillations of the star in time. Such a magnetic field is very strong in the star's atmosphere, where the time-averaged ratio of magnetic pressure to thermodynamic pressure is ~ 1 at optical depth 0.2 and ~ 0.6 at optical depth 0.7. However, the importance of the magnetic field declines rapidly below the photosphere. For this reason, the bulk pulsational properties of the magnetic model resemble closely those for the nonmagnetic prototype.

One small difference is the magnetic field's enhancement of the pulsation amplitudes in the outer layers, where the net effect of the magnetic stresses is to reduce the local effective gravity; the surface velocity amplitude of 103 km s^{-1} should be compared with the nonmagnetic model's 93 km s^{-1}. A comparison of the magnetic and nonmagnetic models is shown in Figure 1.
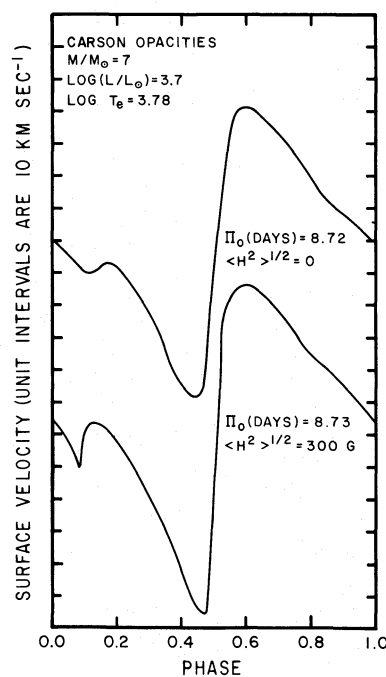

FIG. 1.—Surface velocity curves for full-amplitude models with and without a *uniform* mean magnetic field imposed on the initial equilibrium model.

With our present requirement that magnetic flux be conserved at all layers, the variations of the magnetic field strength follow precisely the variations of the mass density, viz., $\langle H^2 \rangle^{1/2} \propto \rho^{2/3}$. Since the density varies by a factor of ~ 10 at the surface, the magnetic field strength varies there by a factor of ~ 5. Now the surface density attains large maxima at *two* phases: (1) during the brief episode of rising light, when the atmosphere has undergone collapse, but before an outward moving shock wave heats the visible layers; and (2) about half a period after the epoch of maximum light. Interestingly enough, the observed strengths of the varying magnetic fields in two classical Cepheids having periods of 7 days that were studied by Weiss, Dorfi, and Tscharnuter (1981) exhibit maxima at just about these phases. Weiss *et al.* have also established observationally that the polarity of the magnetic field is not reversed during the pulsation cycle, which also agrees with the present theory, although the observed surface field, to be observable, must be more coherent than we have assumed.

IV. THREE MODELS WITH NONUNIFORM MEAN MAGNETIC FIELDS

In order to increase the importance of the magnetic field in the deeper pulsating layers of our models, we have calculated three additional models with a uniform value of $v = 0.5$ applied to the initial equilibrium state. Results for these three models are presented in Table 1 and in Figure 2, where they are compared with results for their nonmagnetic counterparts published in an earlier paper (Vemury and Stothers 1978). For the purpose of easy identification, these nonmagnetic models can be

TABLE 1

FULL-AMPLITUDE PROPERTIES OF THE MODELS WITH AND WITHOUT NONUNIFORM MEAN MAGNETIC FIELDS

PARAMETER	COX-STEWART OPACITIES		CARSON OPACITIES			
	$v = 0$	$v = 0.5$	$v = 0$	$v = 0.5$	$v = 0$	$v = 0.5$
$M/M_\odot$	4	4	7	7	7	7
$\log (L/L_\odot)$	3.503	3.503	3.700	3.700	3.700	3.700
$\log T_e$	3.756	3.756	3.780	3.780	3.720	3.720
$R/R_\odot$	58.7	58.7	65.9	65.9	86.9	86.9
Π_0 (days)	9.75	11.60	8.72	10.35	14.30	17.07
K.E. (10^{42} ergs)	2.4	4.0	8.7	6.4	8.0	13.3
$\Delta R/R$	0.19	0.42	0.19	0.30	0.22	0.35
V_{out} (km s^{-1})	28	58	45	56	37	44
V_{in} (km s^{-1})	-31	-62	-48	-61	-43	-55
ΔV (km s^{-1})	60	120	93	117	80	99
L_{max} (10^{37} ergs s^{-1})	2.0	1.8	2.7[a]	2.4	3.1	2.8
L_{min} (10^{37} ergs s^{-1})	0.79	0.31	0.86	0.61	0.65	0.45
ΔM_{bol}	1.0	1.9	1.2[a]	1.5	1.7	2.0
Asymmetry	4.5	3.3	4.1	2.4	1.5	9.0
ϕ	1.45	0.90	1.59	1.00	1.33	0.88
Π_2/Π_0	0.531	0.461	0.543	0.481	0.505	0.444
Bump	D	A	D	A	A	D

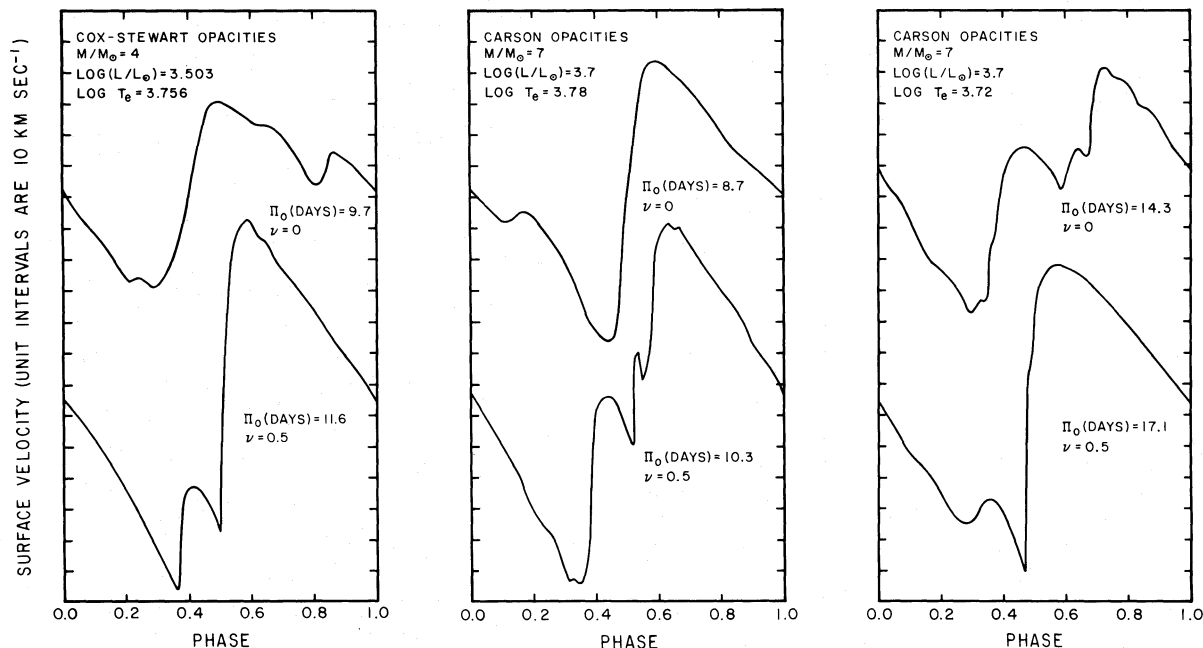[a] Corrected value (cf. Vemury and Stothers 1978).

FIG. 2.—Surface velocity curves for full-amplitude models with and without a *nonuniform* mean magnetic field imposed on the initial equilibrium model. Note the shift of the secondary bump of the velocity curve from one branch to the other when a strong magnetic field is introduced.

described as (1) the prototype bump Cepheid model based on the Carson opacities, (2) a much cooler bump Cepheid model based on the same opacities, and (3) the prototype bump Cepheid model based on Cox-Stewart opacities (the so-called Goddard model).

Interior velocity curves for the prototype bump Cepheid model based on the Carson opacities were published by Vemury and Stothers (1978) in a crude hand-drawn form, but were later shown by Carson, Stothers, and Vemury (1981) in a much-improved machine-generated version. For comparison, Figure 3 displays interior velocity curves for the new magnetic counterpart model. The Christy "echo" phenomenon that produces the secondary bump at the stellar surface can be readily discerned in the magnetic model just as it can in the nonmagnetic model.

As could have been expected from the large reduction of the effective gravities throughout the envelope, the magnetic models have longer periods and larger amplitudes than do the nonmagnetic models. But since the
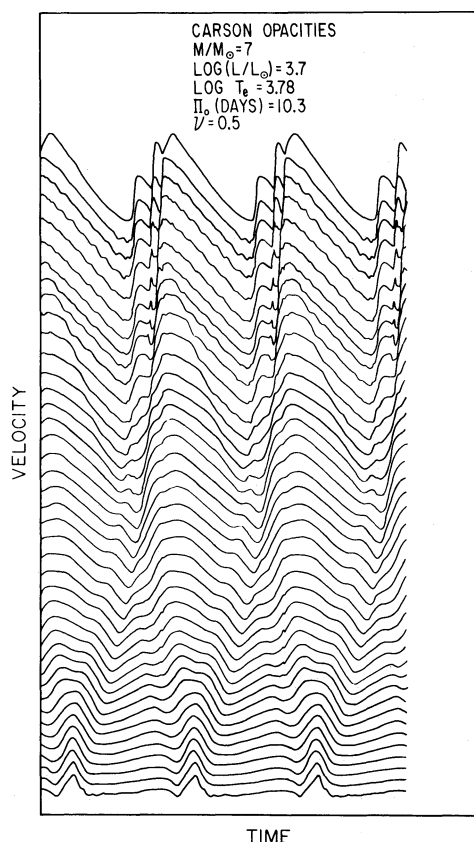
FIG. 3.—Velocity curves for the mass zones in a full-amplitude model with a *nonuniform* mean magnetic field imposed on the initial equilibrium model. The vertical scale is different for the various zones.

FIG. 4.—Relation between the period ratio Π_2/Π_0 and the phase ϕ of the second bump on the surface velocity curve, for theoretical models of nonmagnetic classical Cepheids (*dots*), type II Cepheids (*squares*), RR Lyrae stars (*triangles*), and magnetic classical Cepheids (*open circles*).

overtone periods are less affected by the magnetic field than is the fundamental mode (Stothers 1979b), the period ratios Π_2/Π_0 decrease. Whether there is an important physical link or not, the phases ϕ also show a significant decrease, and therefore seem to obey the "rule" proposed by Simon and Schmidt (1976). The formal correlation between the two quantities is exhibited in Figure 4, to which are added points for a large number of previously published models not only of classical Cepheids but also of type II Cepheids and RR Lyrae stars (Vemury and Stothers 1978; Carson, Stothers, and Vemury 1981; Stothers 1981). (In some cases, unpublished Π_2/Π_0 and ϕ values have been supplied here.) Observe that shifts in this diagram due to the imposition of a magnetic field are qualitatively similar to the shifts that arise from changes of stellar mass, radius, chemical composition, and opacity.

Note, too, that models showing $\phi = 1.4$–1.9 and $\phi < 0.9$ have the secondary bump of the velocity curve appearing on the descending branch, while models showing $\phi = 0.9$–1.4 and $\phi > 1.9$ (extrapolated value) have this bump appearing on the ascending branch. The crucial phase at which the bump crosses velocity maximum—$\phi = 1.4$—corresponds to $\Pi_2/\Pi_0 = 0.51$ or, alternatively, to $Q_0 = 0.046$ days, the correlation between
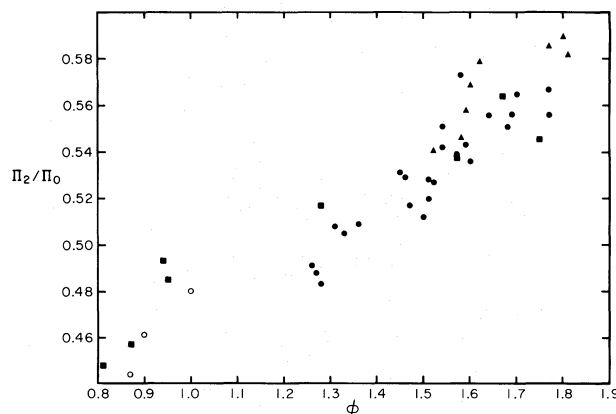
Π_2/Π_0 and $Q_0 = \Pi_0(M/M_\odot)^{1/2}(R/R_\odot)^{-3/2}$ being very good (see, e.g., Fitch 1970). The crossing at velocity minimum takes place both at $\phi = 0.9$ and at $\phi = 1.9$. However, observationally significant bumps seem to occur mainly inside the interval $0.9 < \phi < 1.9$. It would seem, therefore, that linear calculations of the fundamental mode alone should be adequate in most cases to locate the secondary bump of the velocity curve on the descending branch if $Q_0 = 0.036$–0.046 days or on the ascending branch if $Q_0 = 0.046$–0.056 days.

The magnetic models can be used to generalize the (Π_0, M, R) and (Π_0, ϕ, R) relations derived by previous authors for standard, nonmagnetic models. Since the time-averaged march of v is fairly constant throughout the magnetic models, the magnetic effects will arise essentially through a factor $1 + v$, according to equation (1). Therefore we may formally set

$$\Pi_0 = \alpha(1 + v)^\gamma(R/R_\odot)^{7/4}(M/M_\odot)^{-3/4} \qquad (5)$$

and

$$\Pi_0\phi = \beta(1 + v)^\delta(R/R_\odot), \qquad (6)$$

where α, β, γ, and δ are constants to be determined and where we shall express Π_0 in days. For the Cox-Stewart opacities, $\alpha \approx 0.022$ and $\beta \approx 0.25$ (Stobie 1974) while, for the Carson opacities, $\alpha \approx 0.025$ and $\beta \approx 0.22$ (Vemury and Stothers 1978). Independently of which set of opacities is adopted, the models including magnetic fields indicate that $\gamma \approx 0.4$ and $\delta \approx -0.7$. Combination of equations (5) and (6) then yields

$$M/M_\odot \approx \epsilon(1 + v)^{2.2}\Pi_0\phi^{7/3}, \qquad (7)$$

where $\epsilon \approx 0.16$ (Cox-Stewart opacities) or $\epsilon \approx 0.25$ (Carson opacities).

The observed periods and bump phases of the bump Cepheids average $\langle\Pi_0\rangle = 8$ days and $\langle\phi\rangle = 1.6$ (Fricke, Stobie, and Strittmatter 1972), and their masses average $\langle M/M_\odot\rangle = 7$ if a normal evolutionary mass-luminosity relation is assumed (Fricke, Stobie, and Strittmatter 1972; Vemury and Stothers 1978). To satisfy equation (7)

it is necessary to have $v \approx 0.3$ if the Cox-Stewart opacities are used or $v \approx 0.1$ if the Carson opacities are used. (Since the most recent opacities available from the Los Alamos Scientific Laboratory lie between the Cox-Stewart and Carson values, a simple linear interpolation suggests that $v \approx 0.2$ if these opacities are used.) Our present, direct estimates of v supersede our earlier, somewhat larger predicted values of v based on linear computations of Π_2/Π_0 (Stothers 1979b).

A potential problem may arise for magnetic Cepheid models based on these estimated values of v. The theoretical velocity amplitudes are, for both sets of opacities, ~ 100 km s^{-1}, whereas the observed velocity amplitudes, after correction for geometrical projection and for limb darkening, average ~ 60 km s^{-1} (Ledoux and Walraven 1958). Although the model predictions are always worst near the stellar surface, such a discrepancy seems quite large.

V. CONCLUSION

If magnetic fields possessing small-scale chaotic structure are introduced into models of classical Cepheids, the fundamental periods of the models become longer, their amplitudes increase, and the secondary bumps on their velocity and light curves are shifted to earlier phases. Provided that real Cepheids have normal evolutionary masses, the magnetically induced changes of period and of bump phase ϕ certainly point in the direction of better agreement between theory and observations. In fact, exact agreement can be obtained if the average value of the magnetic strength parameter v is taken to be about 0.3 (using the models built with the Cox-Stewart opacities) or about 0.1 (using the models built with the Carson opacities).

On the other hand, significantly larger values of v, say $v \approx 0.8$ (Stothers 1979b), seem to be required if we are to account in a similar way for the puzzlingly low period ratios Π_1/Π_0 seen in double-mode Cepheids. Although the range of fundamental periods observed for double-mode Cepheids, $\Pi_0 = 2-6$ days, does not overlap the range observed for bump Cepheids, $\Pi_0 > 6$ days, it is hard to imagine such a large difference in magnetic field strength separating the two groups of otherwise rather similar Cepheids. An independent method of estimating v exists and relies on empirical periods, radii, and masses (through eq. [5]). But the results obtained in this way (Stothers 1979b) must be regarded as highly provisional, because the empirical radii are still quite uncertain; for example, with the most recently published radii (Sollazzo et al. 1981) v comes out to be effectively zero.

We believe that it may turn out to be possible to find some distribution of interior mean magnetic field that affects Π_1/Π_0 relatively more than Π_0 or ϕ. At the same time, such a distribution ought to yield theoretical amplitudes that are not larger than those actually observed, which is not the case with the present models. If all these observational requirements can be met in a self-consistent way, then it may be possible to infer something more definite about the global properties of the magnetic fields inside classical Cepheids.

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R. Stothers: NASA Institute for Space Studies, Goddard Space Flight Center, 2880 Broadway, New York, NY 10025